Circular motion

Teacher's Guide

UK Space Agency

The UK Space Agency is at the heart of UK efforts to explore and benefit from space. It is responsible for all strategic decisions on the UK civil space programme and provides a clear, single voice for UK space ambitions.

The Agency is responsible for ensuring that the UK retains and grows a strategic capability in the space-based systems, technologies, science and applications. It leads the UK's civil space programme in order to win sustainable economic growth, secure new scientific knowledge and provide benefits to all citizens.

ESA

From the beginnings of the 'space age', Europe has been actively involved in spaceflight. Today it launches satellites for Earth observation, navigation, telecommunications and astronomy, sends probes to the far reaches of the Solar System, and cooperates in the human exploration of space.

Space is a key asset for Europe, providing essential information needed by decision-makers to respond to global challenges. Space provides indispensable technologies and services, and increases our understanding of our planet and the Universe. Since 1975, the European Space Agency (ESA) has been shaping the development of this space capability.

By pooling the resources of 22 Member States, ESA undertakes programmes and activities far beyond the scope of any single European country, developing the launchers, spacecraft and ground facilities needed to keep Europe at the forefront of global space activities.

National Space Academy

Established in 2011 and led by the National Space Centre, the National Space Academy is now the UK's largest space education and skills development programme for secondary and further education. Its team includes some of the country's best science teachers, project scientists and engineers who deliver masterclasses and intensive teacher training for thousands of students and teachers across the UK every year. Internationally the Academy works extensively with the European Space Agency, the UAE Space Agency, and it also leads the UK's ongoing space education and skills development work with China.

Astro Academy Principia

A unique education programme developed by the UK's National Space Academy for the UK Space Agency and ESA (European Space Agency), Astro Academy: Principia uses a suite of demonstrations filmed by ESA astronaut Tim Peake aboard the ISS during his six month Principia mission to explore topics from secondary physics and chemistry curricula. The programme is made up of stand-alone teaching films, downloadable video clips, downloadable files that can be used with the free-to-use dynamical analysis software programme "Tracker", written teacher guides and links to more than 30 further teaching activities.

Principia

Tim's mission to the International Space Station, called 'Principia', used the unique environment of space to run experiments as well as try out new technologies for future human exploration missions. Tim was the first British ESA astronaut to visit the Space Station where he spent six months as part of the international crew.

Circular Motion

Introduction

Tim Peake's mission aboard the International Space Station (ISS) was itself a classic demonstration of centripetal physics – the gravitational attraction of the entire Earth upon Tim's mass is what kept him moving in a nearly circular orbit around the planet for six months. This teacher guide concentrates on classroom demonstrations and orbital experiments aimed at deepening understanding of the modelling of Newton's laws and the role of centripetal forces. The ground-based experiments, focused on qualitative and quantitative understanding of motion in vertical circles, are coupled with Tim's orbital demonstrations in microgravity, which yield very different results. The applications range from aerospace and astronautic training of pilots for the rigours of launch to better understanding the spectacular visual spectacle of the Northern and Southern lights. Classic modelling and extension questions are included to deepen understanding in multiple areas of application.

Tim's orbital demonstrations

Clips:
V1 Ball on tether released - horizontal plane
V2 Ball on tether released – vertical plane

1) Newton's First Law

On Earth, demonstrations of Newton's First Law can sometimes be problematic due to the nature of resistive forces such as friction between an object and a table surface – or confusion can arise as we may be forced to model in one dimension or plane only. Microgravity gives a superb environment to demonstrate, in multiple orientations, the consequences of Newton's First Law.

In this demonstration, we can clearly see the implications of Newton's first Law of Motion. At the start, Tim has a plastic green ball confined to a circular orbital path because he is holding on to the end of the string. The ball's momentum wants to keep it moving in a straight line at a constant speed - but the tension force exerted by Tim through the string and onto the ball is deflecting it from a straight line path. The tension force – directed (from the ball's point of view) inwards towards the centre of the circular path – is providing the role of a centripetal, or centre-directed, force.

Initially, tension in the string provides the centripetal force necessary to force the ball to move with circular motion

When Tim releases the string, the ball moves with a trajectory of constant speed and direction

When Tim releases the string, the tension force collapses. Then, with no centripetal force acting, there is no overall (resultant) force acting on the ball. Since the air resistance is minimal (see later), the ball now has no force acting on it to change its momentum state and so, keeps moving in a straight line at a constant speed - in the direction it was moving at the moment the tension collapsed.

Tim's demonstrations show this in multiple orientations.

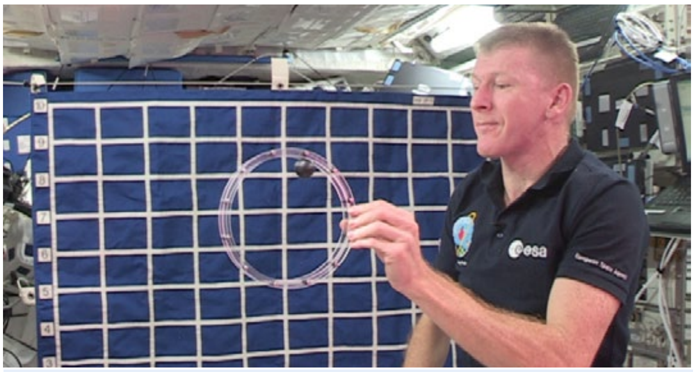
2) Circular Loops

Clips:
V3 Vertical ball in hoop - Tim holding
V4 Horizontal ball in hoop - Tim holding
V5 Vertical ball in hoop - clamped

Circular motion modelling on Earth is often a complex process in which multiple forces which contribute towards a centre-directed, or centripetal force, must be considered. In many cases, the relative contribution of these forces will change during a single complete cycle of modelling whatever is moving in a circle. Microgravity offers an environment in which initial simple modelling can be introduced in a way that's much more challenging here on Earth.

In these demonstrations Tim has set a plastic ball rolling along a polycarbonate loop track. In the microgravity environment of the ISS, once the ball is rolling, the only effective force acting on it is the contact force of the track on the ball – which is directed inwards, towards the centre of the loop. This contact force is therefore providing the centripetal force needed to confine the ball to a circular path.

Because this contact force arises as a result of the ball's motion along the track, there is no minimum force needed to confine the ball to its circular path. As long as the ball continues to roll, the contact force between ball and track will confine it to a circular path and it will keep rolling around the track. In several of the standalone clips, we can see over a matter of minutes that the very low amount of air resistance due to the ball's motion through the atmosphere of the Columbus module does indeed slow it down, but it still completes the loop path it is following. It is only when the ball is dislodged, and the contact force disappears, that the circular motion is replaced by linear motion of the ball moving away from the track and across the laboratory.



The microgravity environment allows a much more complete modelling of Newton's first law and circular motion than on the Earth.

This is in clear contrast to the ground experiment where the interplay between contact force and gravitational force on the ball both contribute to the centripetal force. On Earth, a minimum velocity is needed to complete a vertical loop – but in microgravity this is not the case.

3) Atmospheric drag and air currents in the Columbus module

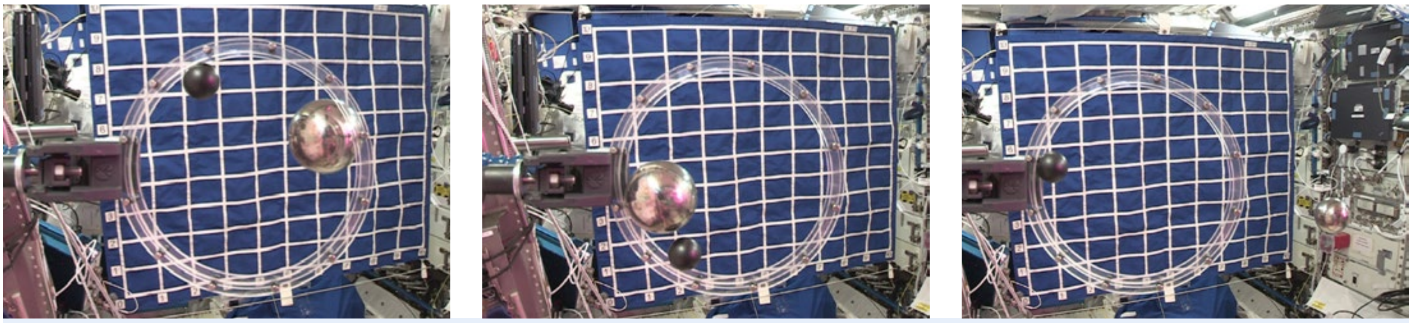
Clips:
V6 Large and small ball in circular track air resistance

In this clip we see Tim has set two balls in motion rolling around the track – the small dense black ball used in previous demonstrations and a much larger, hollow and less massive metal sphere. As expected, both gradually slow down due to air resistance whilst rolling around the track. However, at a very low velocity the smaller, yet greater mass, ball continues to roll around the track whilst the larger hollow sphere, travelling at the same speed, is seen to drift off the track and traverses the Columbus module in a noticeably curving path!

A good extension task would be to invite suggestions from the students as to why this behaviour is observed.

Answer: The microgravity environment on the ISS means that atmospheric convective processes do not occur – instead, air has to be forcibly circulated by fans and blowers. In the Columbus module, the mean air flow is at a speed of around 10cm/s. It is directed towards the rear of the module – where the grid backdrop used for our experiments is located.

As the balls reduced in speed around the track, the the force of the airflow on them remained in the same direction – almost at 90 degrees to their direction of rolling around the track. At higher rolling velocities, the angular momentum of the balls gave them greater stability against the effects of this airflow.



Atmospheric drag and airflow within the ISS produce some interesting effects on the two differently sized balls

But as their speeds reduced over time, it was the lighter (hollow), larger sphere which, due to its larger surface area and lower mass, was more susceptible to being dislodged by the airflow than the smaller more massive ball.

Eventually, the airflow dislodges it from the track. After the larger sphere departed the track, the smaller one was observed to continue its rolling path at even lower speeds.

The larger sphere is observed to drift towards the grid screen and then to change direction back towards the camera – a consequence of the air flow in the Columbus module being partially blocked by the screen and therefore changing direction and carrying the ball with it.

4) Discussion: Forces – are they centripetal or centrifugal?

This is one of the best ways to start what can be a very prolonged discussion/debate/argument amongst physics students, teachers and professionals! A simple distinction is outlined below:

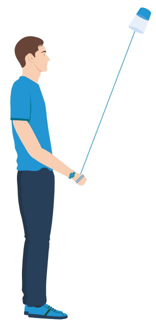
Centripetal forces are real forces which make objects followed curved paths. In simple level modelling, they are directed towards the centre of the circular path instantaneously being followed by the object as a result of that particular force or system of forces being applied.

Centrifugal forces are pseudo-forces that seem to exist within the rotating frame of reference being modelled. It is often "experienced" as a consequence of the reaction, or contact, force produced by the real centripetal force. As an example, anyone who has ridden on a roundabout will know that, from their perspective, it feels as if a force is trying to "fling" them outwards. No actual real force exists that is trying to effect this change.

Ground based experiments

The modelling of how various forces contribute at different stages towards maintaining motion in vertical circles can be done qualitatively or quantitatively – these demonstrations are classic ones which can be conducted as stand-alone activities or after the basics of the role of a single centre-directed force is discussed using Tim's demonstrations.

1) Vertical Cup of Water



Curriculum Links:

- Centripetal forces
- Gravity

Key Stage: 5

Equipment List:

- Paper or polystyrene cup
- 1m length of string
- Scissors and tape to construct cup swing
- Water

Procedure:

To make the swing:
Using scissors, pierce two holes in the cup about 1cm from the top on opposite sides. Thread one end of the string through one hole and tie a knot in it so that it cannot be pulled back through. Thread the other end through the other hole and repeat. Use several pieces of tape to reinforce the area around the holes to prevent the string being ripped out.

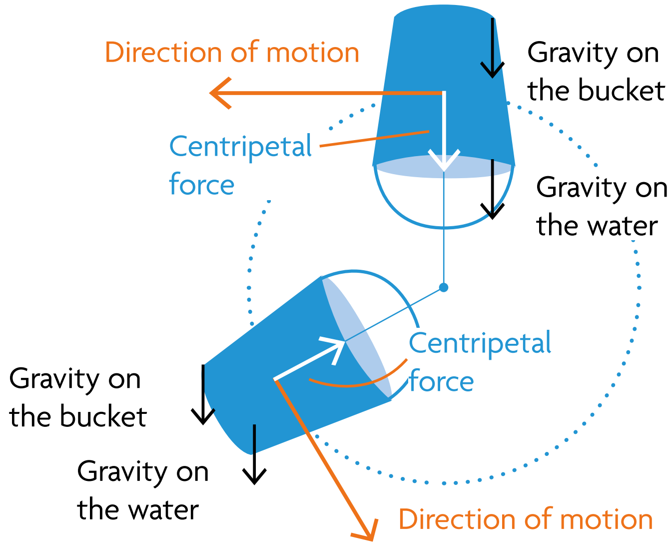
To demonstrate centripetal acceleration:

- Fill the cup about ¼ full with water.
- Starting with the cup hanging down freely, swing it back and forth a few times to build up speed.
- Then, set the cup swinging in a vertical circle by your side.
- After a few rotations, slow down and add some sideways rotation to avoid the water spilling out of the cup as you bring it to a rest.

For an extension exercise you can put a plastic ball into the cup instead of the water and see how slowly you have to loop the cup and ball until the ball falls out of the cup.

Expected Outcomes:

If you turn the cup upside down normally, you will expect the water to fall out under the action of gravity. However, when the cup is swinging in a circular motion, the tension in the string produces a centripetal force on the cup, and therefore a centripetal acceleration.



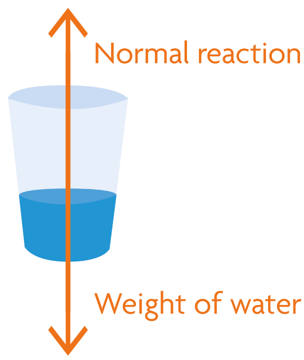
Consider the cup at the bottom of the loop.

The weight points down and the normal force points up; so the net force is their difference. The normal force points toward the centre, so it should be given the positive value. The net force is the centripetal force provided by the tension in the string.

$\sum F = ma$

$N - mg = mv^2/r$

$N = m\,(v^2/r + g)$



At the top of the loop the normal and weight point in the same direction.

$\sum F = ma$

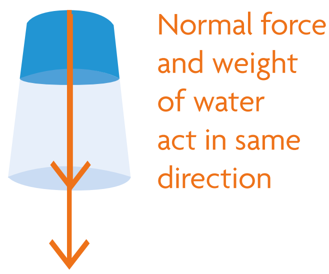
$N + mg = mv^2/r$

$N = mv^2/r - mg$

If the speed of rotation, v, is sufficient for the tension in the string to produce a centripetal acceleration greater than the acceleration due to gravity, i.e.

$v^2/r > g$

then the water will not fall out of the cup, but instead be forced by the reaction force between it and the bottom surface to follow the path of the cup.



2) Vertical Marble Loop-the-Loop

Curriculum Links:

- Centripetal motion
- Gravity
- Reaction/contact forces

Key Stage: 5

Equipment List:

- Loop the loop marble track and marble
  click here for the link for the kit used in the video
- Blu-tack to hold the struts in place
- A long table or laboratory bench

Procedure:

Constructing the marble track
Follow the instructions on the box to fit together the track and build the supports. You will want to use some blu-tack to hold the supports down in place on the table. Use trial and error to judge the necessary height of the lead in track and use a piece of coloured tape to mark the release position for the marble to just make it around the loop.

Demonstrating the loop
Show three outcomes for the loop. Firstly, release the marble a little below the marker tape so that it rolls up the loop but does not complete a loop.

Then, release the marble at the tape to show it just completing a loop and losing contact briefly at the top of the loop.

Finally, release the marble a good way above the marker tape to show it easily completing the loop.

Expected Outcomes:
Ignoring the effects of air resistance, there are two main forces acting on the ball as it rolls around the track:

- the effect of gravity - its weight - which will always be directed downwards
- the reaction, or contact force of the track on the ball itself - which will always be directed inwards and at 90 degrees to the surface of the track

The combination of these two forces provides the centripetal force to keep the ball moving in a circular path.

At the bottom of the loop
The marble is moving with minimum velocity.

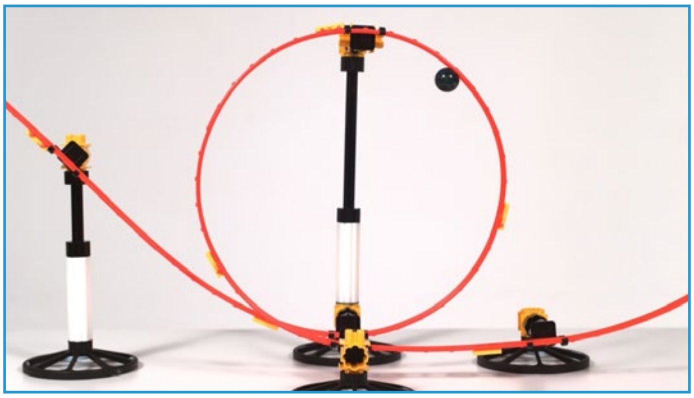
Gravity is directed downwards and the contact force upwards. The resultant force (which acts as the centripetal force) is the contact force (which is upwards or normal) minus the gravitational force (which is downwards).

$\sum F = ma$

$N - mg = mv^2/r$

$N = m\,(v^2/r + g)$

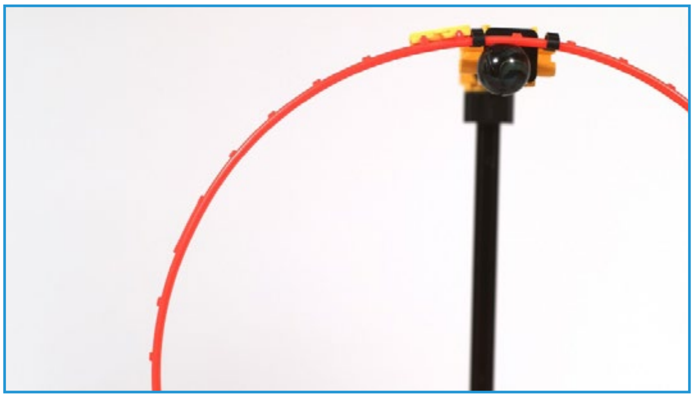
Half way up the loop
The marble is slowing down.



Gravity is still acting downwards but the contact force is at 90 degrees to the track surface - directed towards the centre of the circle.

The resultant centripetal force is purely the contact force.

At the top of the loop



The marble is moving with minimum velocity.

Gravity and contact forces are both directed downwards.

The resultant centripetal force is the contact force plus the gravitational force.

$\sum F = ma$

$N + mg = mv^2/r$

$N = mv^2/r - mg$

The marble will complete the loop as long as there is a contact force. To find the minimum velocity it requires to do this we can model the marble as having just lost contact allowing us to model a scenario where the contact force is zero.

If $N = 0$, then $mv^2/r - mg$

As a result, at this point the centripetal force will be provided by gravity alone and since mass cancels:

$g = v^2/r$

So the minimum velocity required to complete the loop is:

$v = \sqrt{(gr)}$

Space and aerospace contexts

Orbital gymnastics and Skylab – NASA's first space station

Launched in May 1973, Skylab was the first United States space station and remained in orbit until 1979. Although much smaller than the ISS (70 tonnes vs 450 tonnes), Skylab's Orbital Workshop space has a much larger internal diameter (over 6 metres) meaning that the astronauts who participated in the 3 crew missions to the Station in 1973-74 had the opportunity to run around the internal storage lockers using the contact forces between their feet and the lockers to create a centripetal force.

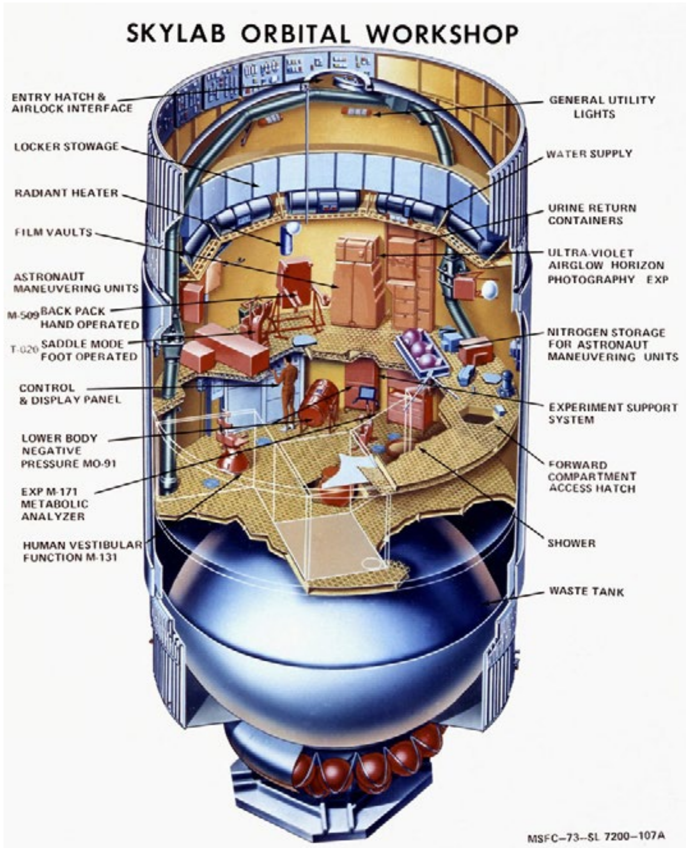
In this clip we see Skylab 2 Commander Pete Conrad running around the lockers: https://www.youtube.com/watch?v=Awe6vOXURpY

Students could measure his period of running (the time taken to complete one circuit) and use the internal station diameter to calculate the effective g-force he was creating in his run.

Alan Bean, the commander of Skylab 3, was a college-level gymnast and the following footage demonstrates many principles of circular motion, conservation of angular momentum about multiple axes and general orbital grace: https://www.youtube.com/watch?v=dmnmuTv4pGE

The crew of the final manned Skylab mission were forbidden to run around the lockers as the vibrations were disrupting solar observations being carried out by the station's automated solar telescopes!

Skylab as seen from the Skylab 4 command and service modules. Credit: NASA

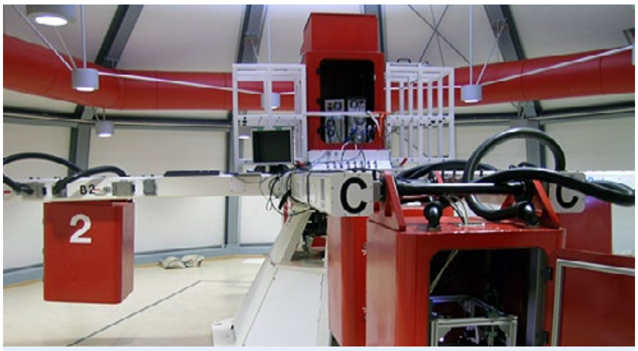


Skylab schematic. Credit: NASA

Centrifuges on Earth for “hypergravity” environments – “Spin your Thesis!”

As seen in the film for this episode, most people are familiar with the use of centrifuges for training pilots, however they are also used by space agencies for generating HYPERGRAVITY environments in which experiments can be conducted.

ESA’s Large Diameter Centrifuge (LDC) at ESTEC (European Space Research and Technology Centre) in Holland can be used to create experimental hypergravity levels from 1 g to 20 g. Automated payloads with a mass of up to 80kg can be installed and there are opportunities for University level students to run research projects there through ESA’s “Spin your Thesis” campaign. http://www.esa.int/Education/Spin_Your_Thesis!_programme



ESA’s Large Diameter Centrifuge at ESTEC. Credit: ESA

Extension work - Generating gravity analogues in space with rotating space stations
Using rotation to generate a gravity analogue in the space environment has been extensively explored in films including 2001: A Space Odyssey, Elysium and, most recently, The Martian.

Although the basic principles are well-understood there are significant opportunities to present challenging extension scenarios for students to explore:


The Space Station from The Martian. Image credit: The Martian, 20th Century Fox

1) In a rotating space station, the effective gravity force experienced is directly proportional to the distance from the axis of rotation. Is there a minimum radius of rotation which must be used to ensure that the differences between what the head and feet experience as gravity pseudo-forces are not harmful to the human body? How would you model this situation?

2) You are in an isolated small room which may or may not be in a rotating space station. There are no windows and you experience what feels like 1 g. The following methods listed would help you determine whether you are on board a space station or not. For each one, QUALITATIVELY describe the differences that would be observed when comparing being on board a rotating space station and being on the Earth for each scenario/experiment:

- Observing the behaviour of a circular pendulum
- Dropping an object vertically.

3) Walking forwards then backwards then left and then right. Repeat again and again. What are the effects on what you experience as your perceived “weight”?

Extension work: charged particles, helical trajectories and the Northern/Southern Lights (aurorae)

Most charged particles from the solar wind which become trapped in Earth's magnetic field will spiral to-and-fro along the paths of the field lines, with the centripetal Lorentz force confining them to orbiting the field lines.

Some of the most energetic solar wind particles trapped in the radiation belts will be funnelled into spiralling paths downwards to impact the Earth's upper atmosphere in polar regions where they produce the auroral displays commonly seen at Northern and Southern latitudes.

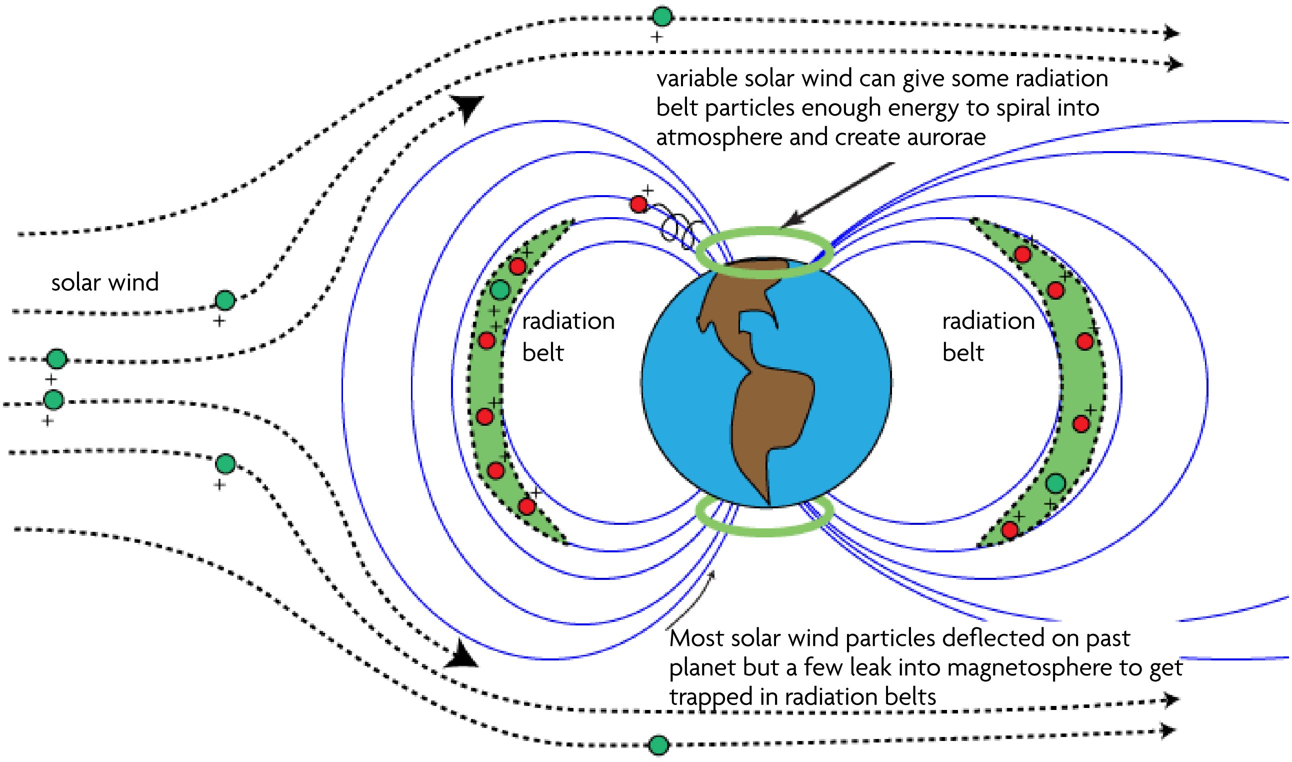
This ESA image shows the Southern aurora – the Aurora Australis, as seen from space by astronauts aboard the International Space Station (ISS) in early 2012.

Tim recorded this unusual footage of the aurorae at sunrise in April 2016: http://www.esa.int/spaceinvideos/Videos/2016/04/Aurora_skimming_the_sunrise

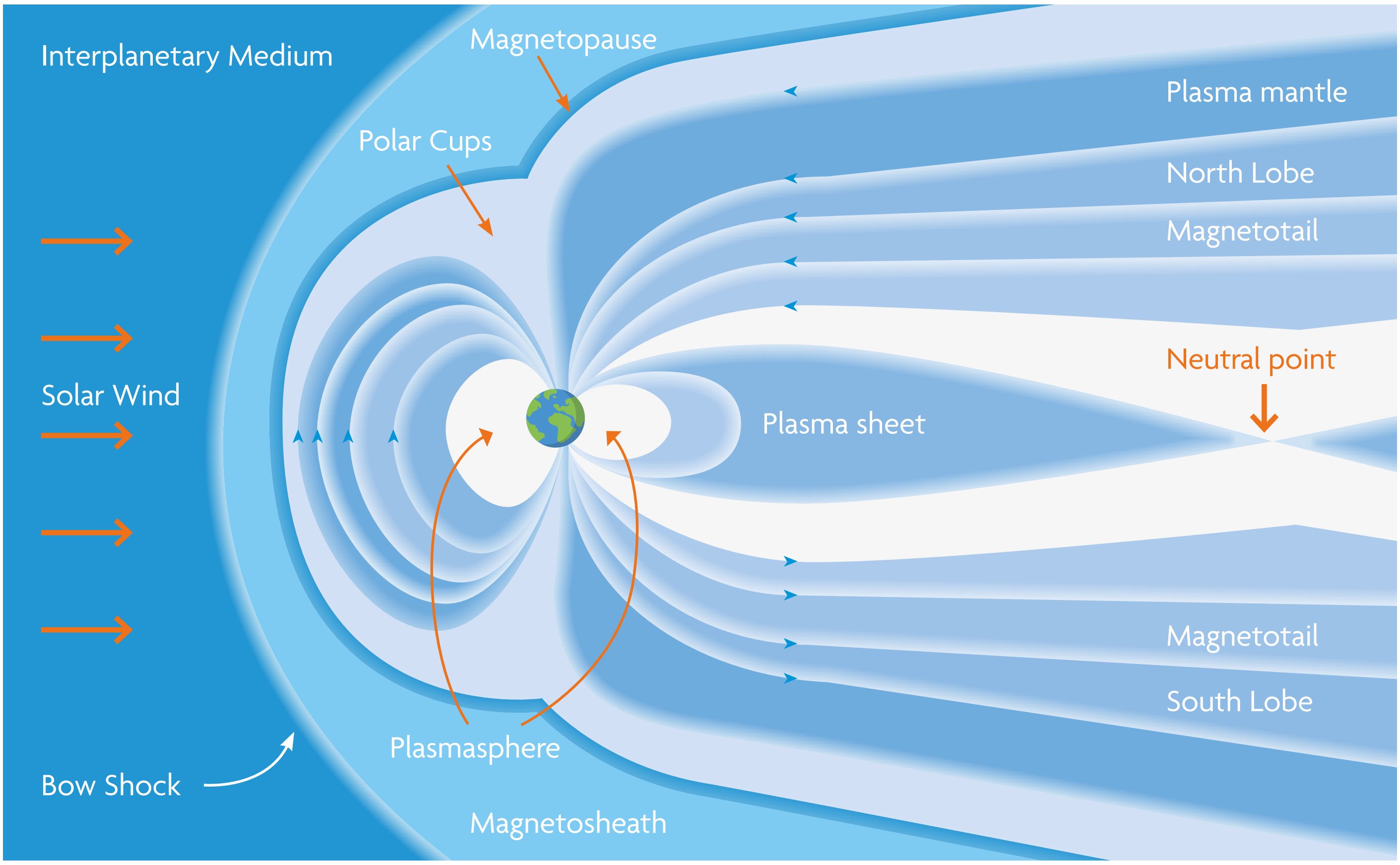
Aurora Australis seen from the ISS. Credit: ESA

Another film of auroral activity filmed by Tim Peake in 2016: http://www.esa.int/spaceinvideos/Videos/2016/04/Aurora_rise

The diagram below summarises the various interactions of the solar wind with Earth's magnetic field. In this diagram, the paths of protons only are shown (electrons are omitted for clarity).



Further features of the Sun-Earth magnetic environment



Explaining the charged particle behaviour
If charged particles, such as a beam of electrons, move through a region with a magnetic field, the external magnetic field will interact with the magnetic field associated with the electron beam and cause a deflection of the electron beam – this is the MOTOR effect.

The resulting force, known as the Lorentz force, is governed by
$F = q(v \times B)$

Note that the force on the charge is derived from the **vector cross product** of v and B – the force produced is at right angles to the component of the velocity that itself is at right angles to the magnetic field vector at that point. The cross product modelling explains the orthogonality of the resultant force with respect to the velocity vector and the magnetic field strength.
$\mathbf{F=Bqv}$

- F = Force in newtons (N)
- q = charge on charge carrier in coulombs (C)
- v = velocity of charge carriers (ms^{-1})
- B = magnetic field strength in tesla (T)

Because of the orthogonal nature of the "motor effect" force, the trajectories of charged particles entering magnetic field regions can be curving around the field lines or "spiralling" along the field lines, as explained below:

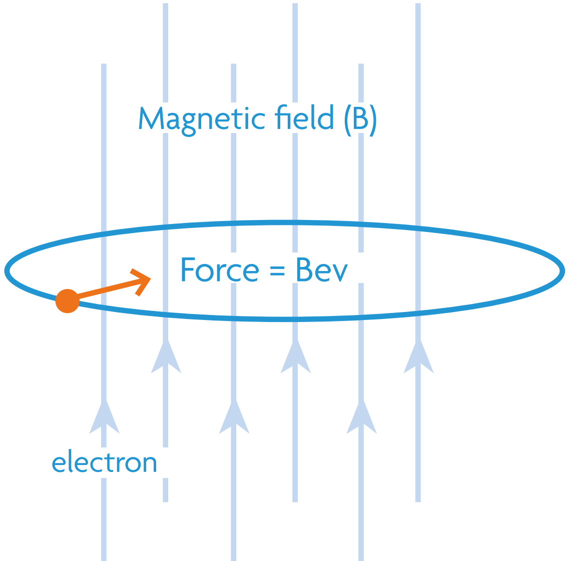
Charged particles orbiting around field lines

Here, the electrons are entering a magnetic field at right angles to the field.

In this case, the force on an electron with charge $e = -1.6 \times 10^{-19}$ coulombs will equal Bev – and this provides a **centripetal force** (centrally directed) confining the electron to a circular orbit.
$Bev = (mv^2)/r$

and so

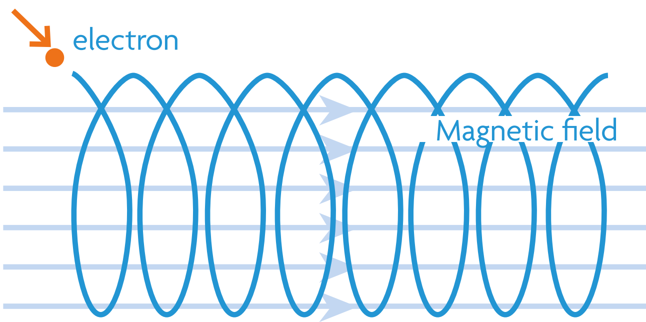
r= radius of orbit = (mv)/Be



This is the principle behind the magnetic confinement of charged particle beams in circular particle accelerators as well as being applicable to the motion of solar wind particles traversing a region of localised magnetic fields.

Charged particles following a helical path around field lines

If the velocity of the charged particles entering the field is not at 90 degrees to the direction of the magnetic field, then:

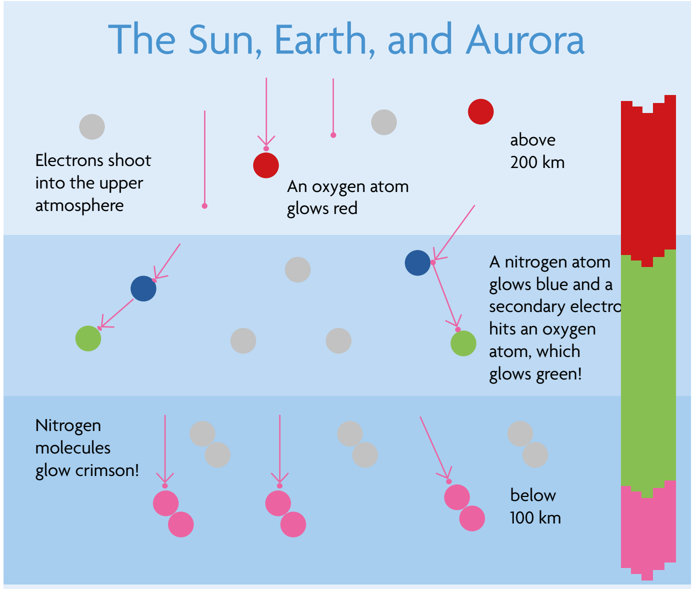


1. The velocity component that is at 90 degrees to the field direction will induce a force on the charged particle. This will cause an orbital effect around the field lines.
2. The velocity component in the direction of the field will remain unchanged. This will cause continuation of lateral translation in the direction of the field lines. There will be no acceleration of the charged particles in this direction since no force will act in this direction.
3. Combining the two effects will lead to a helical path.

This is how the particles trapped in Earth's magnetic field will behave – spiralling to-and-fro along the field lines until some of them interact with the upper atmosphere producing auroral effects.

How are the different colours in Earth's aurorae produced?

There are a variety of charged particles in the Earth's atmosphere due to the various ionic species that exist at different altitudes including nitrogen, oxygen and others. When the charged particles from the solar wind interact with the charged particles in the atmosphere, visible light can be emitted.



The colours of the aurora depend on the composition of Earth's atmosphere at different altitudes

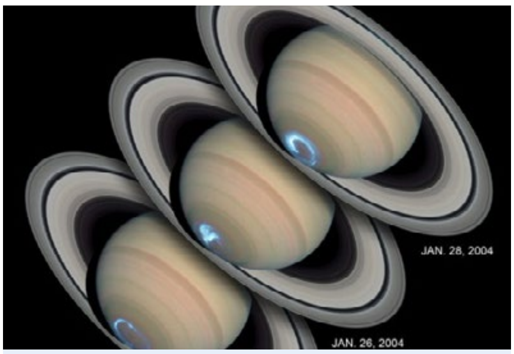
The colours emitted in Earth's atmosphere depend on the relative proportion of oxygen and nitrogen in the atmosphere and the level of solar activity. Colours are also dependent on altitude as the relative abundance of different ionic species will change with altitude.

Atomic oxygen at about 120-180km glows yellow-green but above 322km the colour emitted is red. Nitrogen gives off blue when ionic and red-purple when neutral. The rippling edges are also created by neutral nitrogen.

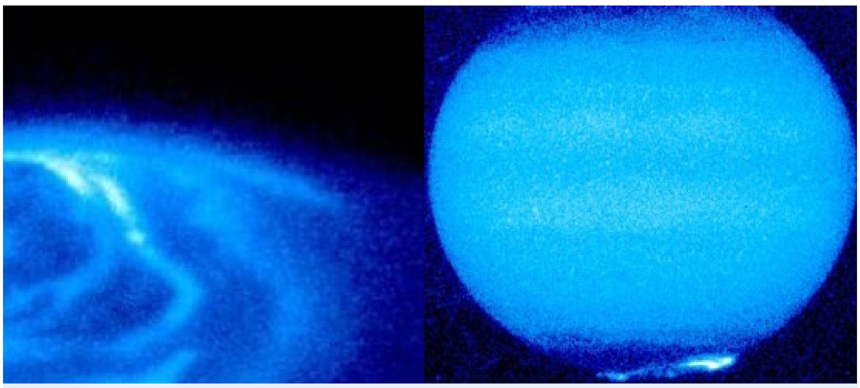
Since most planets in our Solar System have magnetic fields, we would expect those with atmospheres to show auroral features due to solar wind interactions. In these planetary atmospheres, other auroral colours may be seen depending on the constituents of the atmosphere.

Observed features in the atmospheres of Saturn and Neptune are shown in the images below.

The Neptunian magnetic field goes through dramatic changes as Neptune rotates in the solar wind because the magnetic field is tilted 47° from the rotation axis and offset by 0.55 radii from the planet's physical/geometric centre. Due to this unusual orientation auroraoccur over



Aurora on Saturn – montage of imagery from NASA/ESA missions Cassini (Saturn orbiter) and the Hubble Space Telescope (HST)



Neptunian aurorae (HST images) Image courtesy of ESA

1. International Space Station

Determine the orbital speed of the International Space Station - orbiting at 350km above the surface of the Earth. The radius of the Earth is 6.37×10^6m.
($M_{Earth} = 5.98 \times 10^{24}$kg)

2. Human Centrifuge

As part of his preparation for the mission to the International Space Station Tim Peake spent time in a human centrifuge to enable him to cope with experiencing the effects of different g forces.

www.youtube.com/watch?v=D-epG6PWVkI

(a) Tim experienced g-forces of up to 8 g. If the length of the centrifuge arm was 15.0m, at what speed was Tim moving?

(b) In a another space flight simulator an astronaut is rotated horizontally at 20 rpm (revolutions per minute) in a radius arm of length 5.0m. The mass of the astronaut is 75kg.

i. Calculate the centripetal force on the astronaut

ii. Show that this force is equivalent to a gravitational force of about 2.2 g

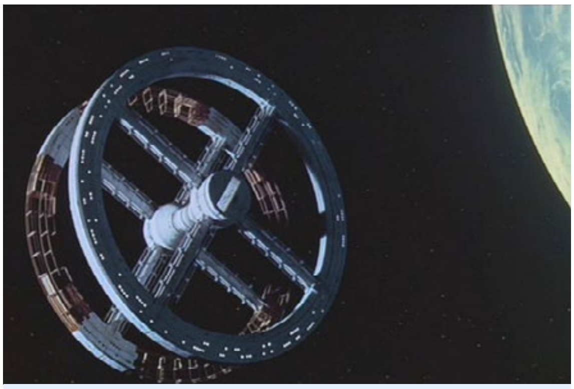
iii. Calculate the rotation rate in rpm that would give a 'simulated' gravity of 3 g

2. Messenger

In 2009, NASA's Messenger spacecraft became the second spacecraft to orbit the planet Mercury. The spacecraft orbited at a height of 125 miles above Mercury's surface. Determine the orbital speed and orbital period of Messenger.
($R_{Mercury} = 2.44 \times 10^6$m; $M_{Mercury} = 3.30 \times 10^{23}$ kg; 1 mile = 1609m)

3. Artificial Gravity and Rotating Space Stations

2001: A Space Odyssey describes three artificial gravity environments, all using rotating bodies: the Space Station in parking orbit around the Earth which uses a huge rotating doughnut; a spinning space lavatory, and the spacecraft Discovery en route to Jupiter which uses a small internal carousel. In the film adaptation, the astronaut Bowman is seen running around this carousel.



http://www.firstshowing.net/img/aot-2001.jpg

(a) To avoid motion sickness the maximum revolutions per minute for a space station is 2.0 rpm (revolutions per minute). Calculate the radius of the space station needed to produce an acceleration of $9.81 ms^{-2}$

(b) For an astronaut of height 2m calculate the difference in acceleration between his head (closer to the axis of rotation) and his feet.

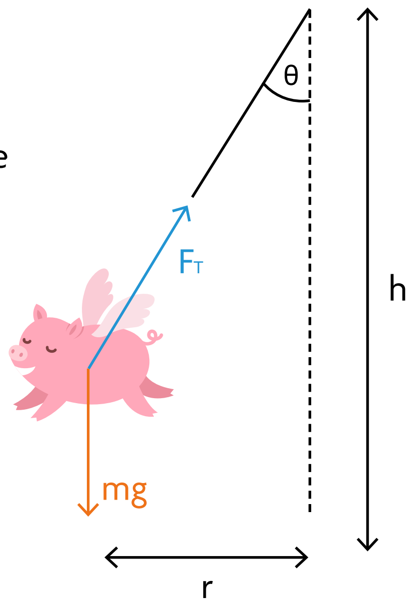
www.steppingoutsidetoobserve.wordpress.com

(c) When travelling to Jupiter, the spaceship Discovery had a special section with a slowly rotating drum to produce an artificial gravity roughly equal to that of the Moon rather than the Earth ($g = 1.7 ms^{-2}$). If the drum makes one revolution every 10 seconds, calculate the radius of the drum.

(d) Calculate the difference between the acceleration of the 2m tall astronaut's head and feet in the Discovery craft. How does this compare with that in the space station?

5. Flying Pigs

You can determine the acceleration due to gravity by finding the time period of a flying pig moving in a banked circular path.



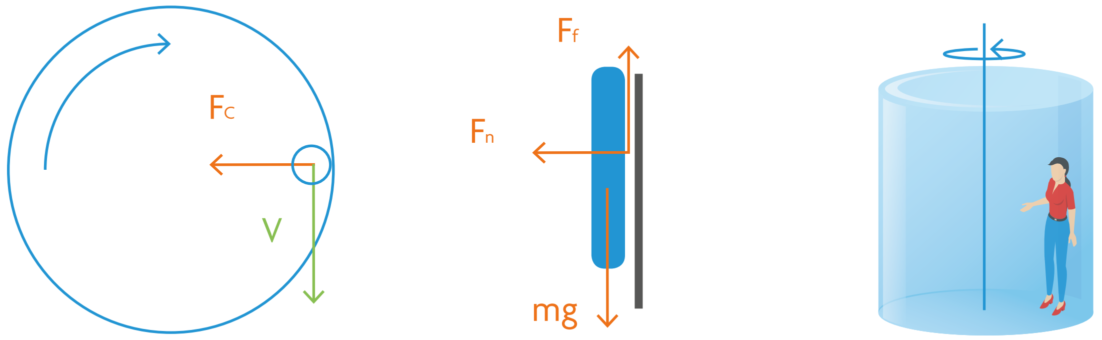
(a) By considering the forces acting on the flying pig as shown in the accompanying diagram, derive an expression for the time period, T, of the pig's orbit in terms of its radius, r, and angle, θ.

(b) Given that $g = 9.81 ms^{-2}$ determine the time period of a pig for a radius of 20.0cm and angle of 30°.

(c) Devise an experiment using this idea where g can be calculated from the gradient of a graph. What will you plot on the x and y axes? How is the gradient related to g?

6. Wall of Death

An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away. The coefficient of static friction between the person and the wall is μ_s, and the radius of the cylinder is R.



(a) Show that the maximum period of revolution necessary to keep the person from falling is:

$$T = \sqrt{\frac{4\pi^2 R \mu_s}{g}}$$

(b) Obtain a numerical value for T if R = 4.00m and $\mu_s = 0.40$

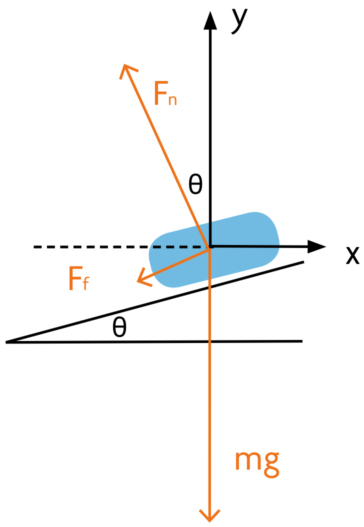
(c) How many revolutions per minute does the cylinder make?

7. Banked Track Circular Motion With Friction

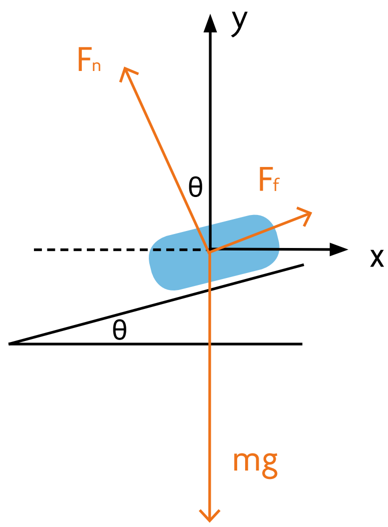
A car rounds a banked curve as in the diagrams below. The radius of curvature of the road is R, the banking angle is θ and the coefficient of static friction is μ.

(a) Determine the range of speeds the car can have without slipping up or down the road when the car is about to slide "up" the bank and when the car is about to slide "down" the bank.

HINT: Friction always opposes the motion so we must look at two possibilities for the friction force. We must use separate free-body diagrams for the two cases



when the car is about to slide "up" the bank



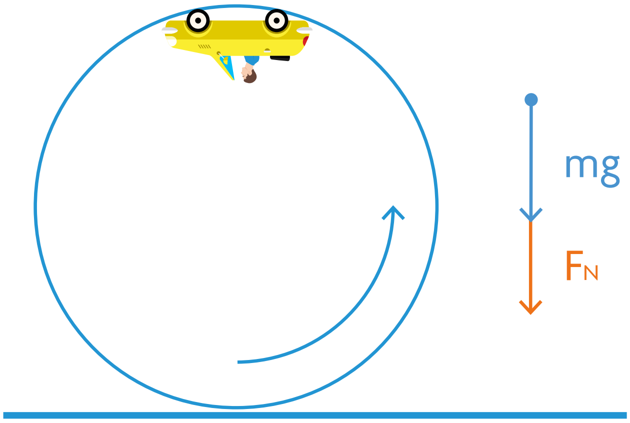
when the car is about to slide "down" the bank

(b) Find an equation for the minimum value for μ such that the minimum speed is zero.

(c) What is the range of speeds possible if R = 100m, θ = 10°, and μ= 0.10 (slippery conditions)?

8. Vertical Circular Motion

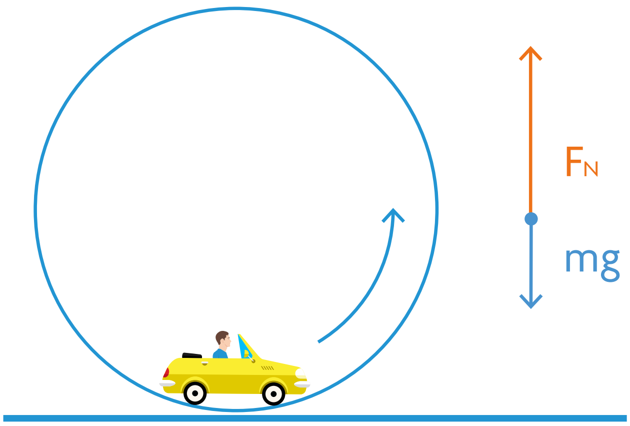
A roller coaster is going through a loop that has a radius of 4.80m. The roller coaster cars have a speed of $13.8ms^{-1}$ at the top of the loop. During testing and development of the roller coaster, it was determined that the cars and passengers have a combined mass of 4800kg on an average run.



(a) Determine the amount of force the track must be designed to withstand at the top in order to keep the cars going around the loop.

(b) Determine the minimum speed the cars on this roller coaster can move in order to just barely make it through the loop at the top.

(c) The track for the roller coaster mentioned in the last two examples needs to actually be stronger at the bottom of the loop. Although the cars will actually speed up as they come down to the bottom of the loop, assume the same velocity, radius, and mass as above and determine the amount of force the track must be able to withstand at the bottom of the loop.



1. Launching a Rocket

Determine the orbital speed of the International Space Station - orbiting at 350km above the surface of the Earth. The radius of the Earth is 6.37×10^6m.
($M_{Earth} = 5.98 \times 10^{24}$kg)
$mv^2/r = GMm/r^2$ so $v = \sqrt{(GM/r)}$
$v = ((6.67 \times 10^{-11} \times 5.98 \times 10^{24}) / (6.37 \times 10^6 + 350 \times 10^3))^{1/2} = 7700 ms^{-1}$

2. Human Centrifuge

As part of his preparation for the mission to the International Space Station Tim Peake spent time in a human centrifuge to enable him to cope with experiencing the effects of different g forces.

www.youtube.com/watch?v=D-epG6PWVkI

(a) Tim experienced "g-forces" of up to 8 g. If the length of the centrifuge arm was 15.0m, at what speed was Tim moving?
$a = v^2 / r$
$v = (8 \times 9.8 \times 15)^{1/2} = 34.3 ms^{-1}$

(b) In a another space flight simulator an astronaut is rotated horizontally at 20 rpm (revolutions per minute) in a radius arm of length 5.0m. The mass of the astronaut is 75kg.

i. Calculate the centripetal force on the astronaut
$\omega = \theta/t$
$\omega = (20 \times 2\pi) / 60 = 2.09 \text{ rads}^{-1}$
$F = mr\omega^2$
$F = 75 \times 5 \times 2.09^2 = 1640N$

ii. Show that this force is equivalent to a gravitational force of about 2.2 g
$F_{grav} = 75 \times 9.8 \times 2.2 = 1620N$ alternative solution $(1640/75) / 9.8 = 2.2$ g

iii. Calculate the rotation rate in rpm that would give a 'simulated' gravity of 3 g
$a = r\omega^2$

$\omega = (3 \times 9.8 / 5)^{1/2} = 2.42 \text{ rads}^{-1}$
$\omega = \theta / t$
$2.42 = (\text{revs} \times 2\pi) / 60$ so revs = 23.1 rpm

3. Messenger

In 2009, NASA's Messenger spacecraft became the second spacecraft to orbit the planet Mercury. The spacecraft orbited at a height of 125 miles above Mercury's surface. Determine the orbital speed and orbital period of Messenger.
($R_{Mercury} = 2.44 \times 10^6$m; $M_{Mercury} = 3.30 \times 10^{23}$kg; 1 mile = 1609m)

$v = \sqrt{(GM/r)}$

$r = 2.44 \times 10^6 + 125 \times 1609 = 2.64 \times 10^6$m

$v = ((6.67 \times 10^{-11} \times 3.3 \times 10^{23}) / 2.64 \times 10^6)^{1/2} = 2890 ms^{-1}$

$v = 2\pi r/T$

$T = 2 \times \pi \times 2.64 \times 10^6 / 2890 = 5740s$

4. Artificial Gravity and Rotating Space Stations

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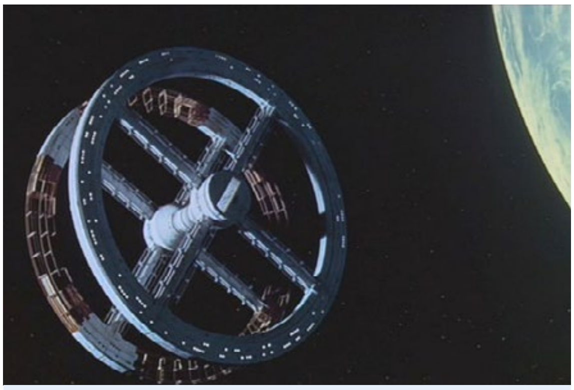

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(a) To avoid motion sickness the maximum revolutions per minute for a space station is 2.0 rpm (revolutions per minute). Calculate the radius of the space station needed to produce an acceleration of $9.81 ms^{-2}$

$\omega = \theta/t$

$\omega = (2 \times 2\pi) / 60 = 0.21 rads^{-1}$

$a = r\omega^2$

$r = 9.81 / 0.21^2 = 222m$

(b) For an astronaut of height 2m calculate the difference in acceleration between his head (closer to the axis of rotation) and his feet.

$a = r\omega^2$

$r = 222 - 2 = 220m$

$a = 0.21^2 \times 220 = 9.70 ms^{-2}$

So difference $= 9.81 - 9.70 = 0.11 ms^{-2}$

(c) When travelling to Jupiter, the spaceship Discovery had a special section with a slowly rotating drum to produce an artificial gravity roughly equal to that of the Moon rather than the Earth ($g = 1.7ms^{-2}$). If the drum makes one revolution every 10 seconds, calculate the radius of the drum.

$\omega = \theta / t$

$\omega = 2\pi / 10 = 0.63 \text{ rads}^{-1}$

$a = r\omega^2$

$r = 1.7 / 0.63^2 = 4.3m$

(d) Calculate the difference between the acceleration of the 2m tall astronaut's head and feet in the Discovery craft. How does this compare with that in the space station?

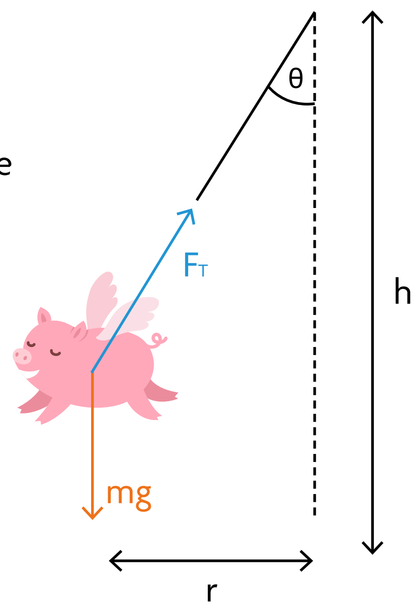
$r = 4.3 - 2 = 2.3m$

$a = 0.632 \times 4.3 = 0.91ms^{-2}$

So difference = $1.7 - 0.91 = 0.79ms^{-2}$ i.e. 7 x bigger than the space station

5. Flying Pigs

You can determine the acceleration due to gravity by finding the time period of a flying pig moving in a banked circular path.



(a) By considering the forces acting on the flying pig as shown in the accompanying diagram, derive an expression for the time period, T, of the pig's orbit in terms of its radius, r, and angle, θ.

Resolving vertically gives $F\cos\theta = mg$

Resolving horizontally gives $F\sin\theta = mr\omega^2$

Dividing these equations gives $\tan\theta = r\omega^2/g$

But $\omega^2 = 2\pi/T$

So $4\pi^2/T^2 = \tan\theta \times g/r$

$T = 2\pi\sqrt{(r/g\tan\theta)}$

(b) Given that $g = 9.81ms^{-2}$ determine the time period of a pig for a radius of 20.0cm and angle of 30°.

$T = 2\pi\sqrt{(0.20 / 9.81 \tan 30)} = 1.18s$

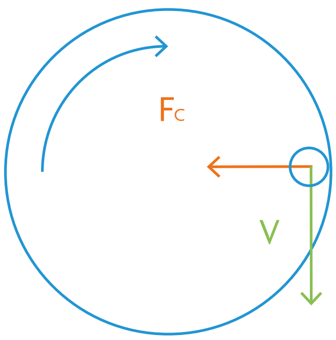
(c) Devise an experiment using this idea where g can be calculated from the gradient of a graph. What will you plot on the x and y axes? How is the gradient related to g?
In a practical experiment r is not as easy to measure as the length of the pendulum L, and as $L\sin\theta = r$, so our equation becomes

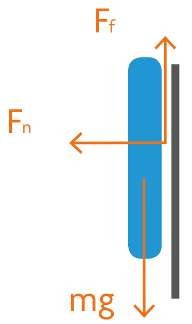
$T = 2\pi\sqrt{(L\sin\theta/g\tan\theta)}$
And so $T = 2\pi\sqrt{(L\cos\theta/g)}$

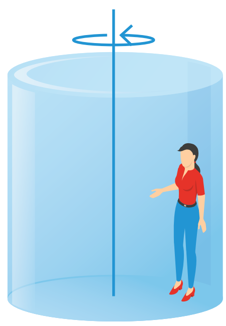
Plotting a graph of T^2 against $\cos\theta$ gives a straight line with gradient $4\pi^2/g$

6. Wall of Death

An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away. The coefficient of static friction between the person and the wall is μ^s, and the radius of the cylinder is R.







(a) Show that the maximum period of revolution necessary to keep the person from falling is:

$$T = \sqrt{\frac{(4\pi^2 R\mu_s}{g)}}$$

$F_n = mR\omega^2$ where $\omega = 2\pi/T$
$F_f = mg = \mu_s F_n$
$mg/\mu_s = mR4\pi^2/T^2$
$T = \sqrt{(4\pi^2 R\mu_s/g)}$

(b) Obtain a numerical value for T if R = 4.00m and $\mu_s = 0.40$
$T = \sqrt{(4\pi^2 \times (4 \times 0.40 / 9.8))} = 2.54$s

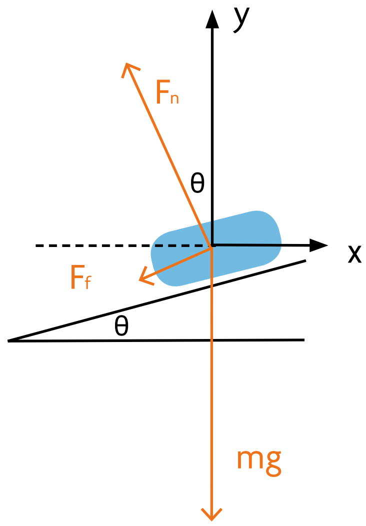
(c) How many revolutions per minute does the cylinder make?
60 / 2.54 = 23.6 revolutions per minute

7. Banked Track Circular Motion With Friction

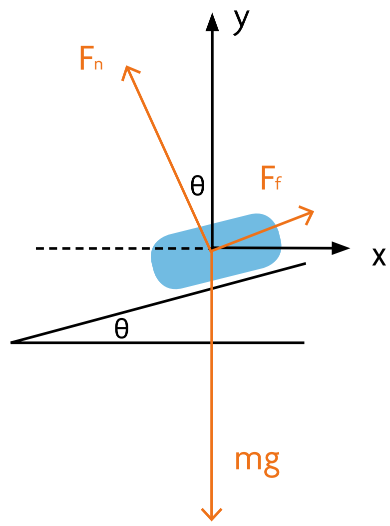
A car rounds a banked curve as in the diagrams below. The radius of curvature of the road is R, the banking angle is θ and the coefficient of static friction is μ.

(a) Determine the range of speeds the car can have without slipping up or down the road when the car is about to slide "up" the bank and when the car is about to slide "down" the bank.

HINT: Friction always opposes the motion so we must look at two possibilities for the friction force. We must use separate free-body diagrams for the two cases



when the car is about to slide "up" the bank



when the car is about to slide "down" the bank

1. max velocity

Resolving horizontally $mv^2 / r = F_n \sin\theta + \mu F_n \cos\theta$ so $v^2 = \frac{r}{m}(F_n \sin\theta + \mu F_n \cos\theta)$

Resolving vertically $mg = F_n \cos\theta - \mu F_n \sin\theta$ so $m = (F_n \cos\theta - \mu F_n \sin\theta) / g$

This gives $v^2 = rg (F_n \sin\theta + \mu F_n \cos\theta) / (F_n \cos\theta - \mu F^n \sin\theta)$

So $v_{max} = \sqrt{(rg (\sin\theta + \mu\cos\theta) / (\cos\theta - \mu\sin\theta))}$

2. min velocity

Resolving horizontally $mv^2 / r = F_n \sin\theta - \mu F_n \cos\theta$ so $v^2 = \frac{r}{m}(F_n \sin\theta - \mu F_n \cos\theta)$

Resolving vertically $mg = F_n \cos\theta + \mu F_n \sin\theta$ so $m = (F_n \cos\theta + \mu F_n \sin\theta) / g$

This gives $v^2 = rg (F_n \sin\theta - \mu F_n \cos\theta) / (F_n \cos\theta + \mu F_n \sin\theta)$

So $v_{min} = \sqrt{(rg (\sin\theta - \mu\cos\theta) / (\cos\theta + \mu\sin\theta))}$

(b) Find an equation for the minimum value for μ such that the minimum speed is zero.

From above, with $v_{min} = 0$ $\sin\theta = \mu\cos\theta$ so $\mu = \tan\theta$

(c) What is the range of speeds possible if R = 100m, θ = 10°, and μ= 0.10 (slippery conditions)?

$v_{max} = \sqrt{(100 \times 9.81 (\sin 10 + 0.1 \times \cos 10) / (\cos 10 - 0.1 \times \sin 10))}$

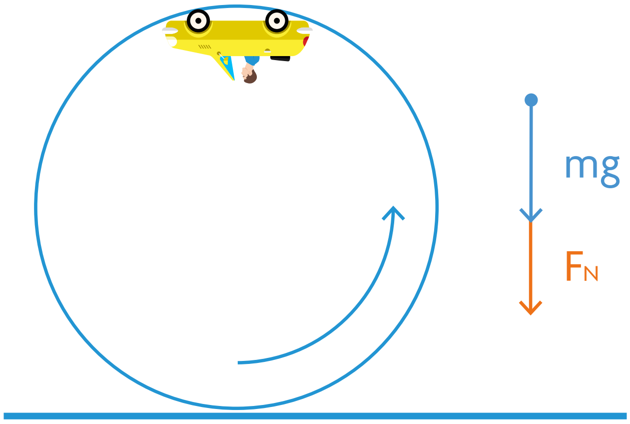
$v_{max} = 16.6 \text{ ms}^{-1}$

$v_{min} = \sqrt{(100 \times 9.81 (\sin 10 - 0.1 \times \cos 10) / (\cos 10 + 0.1 \times \sin 10))}$

$v_{min} = 8.73 \text{ms}^{-1}$

8. Vertical Circular Motion

A roller coaster is going through a loop that has a radius of 4.80m. The roller coaster cars have a speed of 13.8ms^{-1} at the top of the loop. During testing and development of the roller coaster, it was determined that the cars and passengers have a combined mass of 4800kg on an average run.



(a) Determine the amount of force the track must be designed to withstand at the top in order to keep the cars going around the loop.

$F = mv^2 / r - mg = 4800 \times 13.8^2/4.8 - 4800 \times 9.81 = 143{,}000N$

(b) Determine the minimum speed the cars on this roller coaster can move in order to just barely make it through the loop at the top.

$F = mv^2 / r - mg$ where $F = 0$

So $v = (rg)^{1/2} = (4.8 \times 9.81)^{1/2} = 6.86ms^{-1}$

(c) The track for the roller coaster mentioned in the last two examples needs to actually be stronger at the bottom of the loop. Although the cars will actually speed up as they come down to the bottom of the loop, assume the same velocity, radius, and mass as above and determine the amount of force the track must be able to withstand at the bottom of the loop.

$F = mv^2 / r + mg = 4800 \times 13.8^2 / 4.8 + 4800 \times 9.81 = 238{,}000N$